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SUPERSONIC FLOW IN THE AREA OF ANTISYMMETRIC THIN CRUCIFORM WIN--ETC(U)
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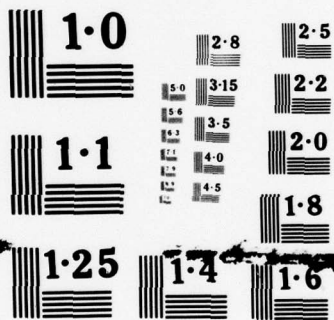
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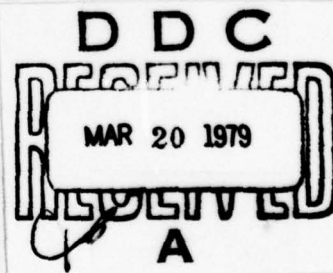
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SUPERSONIC FLOW IN THE AREA OF ANTISYMMETRIC THIN CRUCIFORM
WINGS WITH SUPERSONIC LEADING EDGES IN A HORIZONTAL PLANE,
WITH CONSIDERATION OF FLOW SEPARATION ON THE EDGES

By

Stefan Staicu



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SUPERSONIC FLOW IN THE AREA OF ANTISYMMETRIC THIN **CRUCIFORM WINGS WITH**
 SUPERSONIC LEADING EDGES IN A HORIZONTAL PLANE, WITH CONSIDERATION OF
 FLOW SEPARATION ON THE EDGES

Author: Stefan Staicu

1. General Considerations

A study is made of the flow in the supersonic regime in the area of a thin cruciform wing with an antisymmetric distribution of incident angles. The horizontal plane has supersonic leading edges and flow separation is considered along the subsonic leading edge line of the vertical plane.

We shall therefore consider a cruciform wing **composed** of two simple delta wings perpendicular to one another, and refer it to a system of Cartesian axes $Ox_1x_2x_3$ with an origin in the wing apex and with the axis Ox_1 in the direction of the unperturbed tream U_∞ (figure 1).

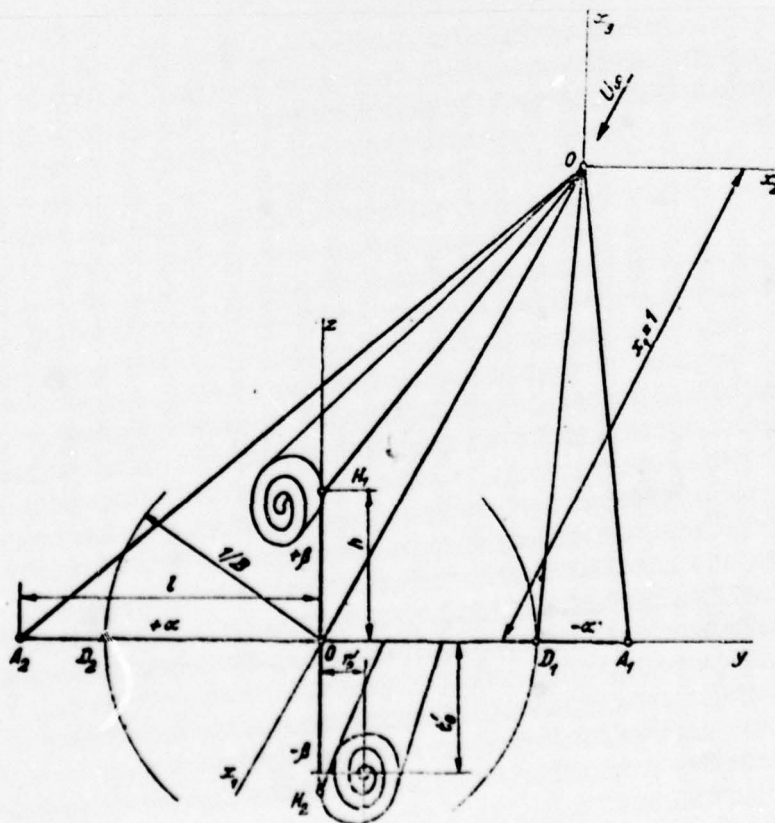


Figure 1

Let α be the equal incidence angles of **opposite** signs along the two halves of the horizontal wing, $+\beta$ the incident angles for the upper part of the sheet and $-\beta$ for the lower part.

Analogous to what happens for a thin cruciform **wing** with complete subsonic edges, in the present case the flow separates at the subsonic leading edges of the sheets, producing a vortex system located on the right and left sheets as a function of the incident angles α and β , which produces antisymmetric motion.

Thus the flow is modified by the existence of two vortex apices **situated** antisymmetrically with regard to the axis of symmetry and with the same intensity and sign (figure 1). Below we shall denote by $(-r'_0, t'_0)$ the coordinates for the physical plane of the vortex core under which the **apex** formed at the edge of the sheet is concentrated.

Since the study of flow is becoming more and more complicated, we shall try below to find the effects of these vortices on the wing and on the sheet. Therefore we shall assume that the effect of flow separation at the leading edges of the sheet and the formation of vortex cores is to create a complex field of vertical and horizontal velocities which will **modify** the flow in such a way that the velocities are zero at the edge.

Under these considerations the flow in the area of the cruciform system remains conical longer and can be treated using the methods of conical flow theory for wings [1].

Proceeding as if the cruciform wing had complete subsonic edges, we shall consider that an actual thin cruciform **wing**, which in a **way** has certain finite velocities at the edges because of the effect of stream separation, is equivalent, from an aerodynamic point of view, to a fictitious thin wing with a convenient variety of incident angles (or **velocities** of lateral perturbation). In order to study the motion with ease, with the **method** of conical motion, we shall divide the fictitious thin wing into three **component** wings.

1. The thin cruciform wing has antisymmetric incidence on the sheets and is thus variable, so that there is some pressure modification and of normal perturbation velocities on the sheet in the vicinity of the leading edge. A fictitious thin wing is obtained with a finite velocity even at the subsonic edges of the plates, but equal and of opposite sign on the two sides, which does not agree with experimental findings.

2. A cruciform wing of "symmetric thickness" with equal sheet slopes and with the same signs as the first incidents of the component wing. This wing, combined with the first, will form a tall cruciform wing which will have different pressures on the two halves of the sheet, somewhat approaching the real situation.

3. A third wing will have a "symmetric thickness" with variable slope, so that in combination with the second wing a mean slope of zero will be found, corresponding to an actual thin wing. This wing will have the role of total compensation for the aerodynamic effects of the second wing in the field of normal perturbation velocities.

If we superimpose these three component fictitious wings, we shall actually be totalling their aerodynamic effects. We obtain a cruciform wing equivalent to a real one, but with consideration of separation on the edge.

2. Axial Perturbation Velocities

Below we shall determine the axial perturbation velocities for these three fictitious component cruciform wings necessary to calculate the pressure distribution on these four arms as though we were determining the aerodynamic properties of real wings. We shall now indicate the flow variables and the planes used.

Starting from the physical plane yOz (figure 1) for the coordinates

$$y = \frac{x_2}{x_1}, \quad z = \frac{x_3}{x_1}, \quad (1)$$

and by transformation

$$\eta = \frac{y}{1 - B^2 z^2}, \quad \zeta = \frac{z \sqrt{1 - B^2 (y^2 + z^2)}}{1 - B^2 z^2}, \quad (B = \sqrt{M_\infty^2 - 1}), \quad (2)$$

we get the auxiliary plane

$$x = \eta + i\zeta, \quad (3)$$

represented in figure 2.

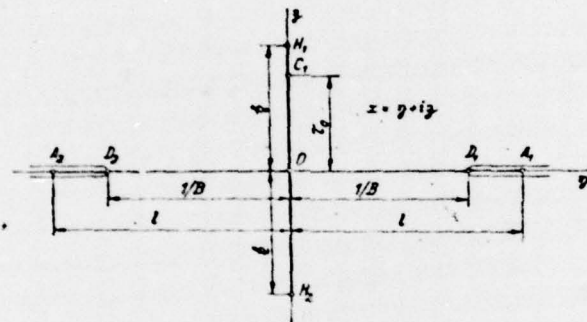


Figure 2.

Starting with figure (2), the height of the sheet and the ordinate of the **core** position of the vortex created at the leading edge of the sheet, x will be given in the auxiliary plane by the following formulae:

$$\eta = \frac{h}{\sqrt{1 - B^2 h^2}}, \quad \zeta = \frac{l_0}{\sqrt{1 - B^2 l_0^2}}. \quad (4)$$

From plane x we shall plot X in the complex plane (figure 3) through suitable transformation

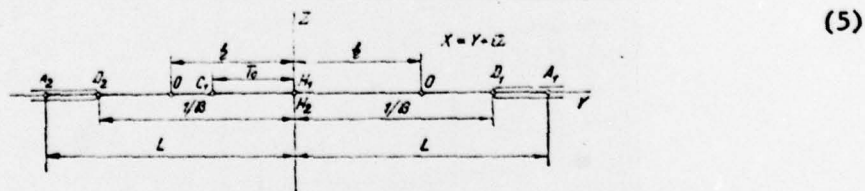


Figure 3.

which is situated in a horizontal plane on the cruciform wing, similar to a plane delta wing. However, in order to define the axial perturbation velocities in primary form, we shall apply the method of hydrodynamic analogy in the plane χ (figure 4), defined by the corresponding transformation

$$x^* = \frac{1 + \Re X}{1 - \Re X}. \quad (6)$$

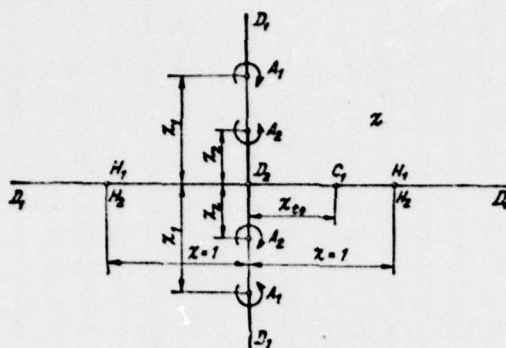


Figure 4.

1. Antisymmetric Thin Cruciform Wing with Variable Incidence

As a result of the effects of these two antisymmetric vortices, the normal perturbation velocities at the surface of the cruciform wing are modified to correspond to a real wing, and therefore with the fictitious thin wing defined above. We shall consider that the vertical velocity on the wing with the supersonic edges is not modified, because of the existing vortices, but that the lateral one on the sheet v' will have the following variations:

$$v_0 = -\beta_0 U_\infty, (y=0, -t_0 \leq z \leq t_0). \quad (7a)$$

$$v' = v'(x) = -\beta'(z) U_\infty, [z=0, y \in (-h, -t_0) \cup (h, t_0)], \quad (7b)$$

such that the velocity at the sheet edges becomes

$$v_1 = -\beta_1 U_\infty. \quad (8)$$

$2h$ represents the opening of the sheet and t_0 is a coordinate which limits the interval of variation of the lateral velocity on the sheet.

The continual variation in velocities v' or of the respective incidence angles corresponds to continuous distribution of elementary edges situated on the surface within the interval under consideration, while their contributions in the expression of axial perturbation velocity from point χ in plane (6) will be

$$d u_l = q_l(\chi_l) \left(\ln \frac{\chi - \chi_l}{\chi + \chi_l} - \ln \frac{\chi - \frac{1}{\chi_l}}{\chi + \frac{1}{\chi_l}} \right) d \chi_l. \quad (9)$$

in which χ'_t is the abscissa of the edge stream in the wake of the wing:

$$\chi'_t = \pm \sqrt{\frac{1 + \mathfrak{K} T}{1 - \mathfrak{K} T}}, \quad T^2 = \mathfrak{b}^2 - \tau^2. \quad (10)$$

Below we shall use the source distribution on the wing $q'_1(\chi'_t)$ as the most simple, corresponding to the conditions imposed by the problem on lateral velocity v' . Thus, looking in plane X, the intensity of the source located in the wake of the sheet will be

$$q'_t(T) = q_t \frac{T}{b}, \quad T \in (-T_0, T_0). \quad (11)$$

Taking this distribution of sources into consideration and applying the formulae established in conical motion theory [1], the axial perturbation velocity of the first component wing will be obtained by adding up the contributions of all elementary distributed edges. Thus, proceeding in plane X through transformation (6), where we determine the contribution of the subsonic edges of the sheets and the supersonic ones of the horizontal planes, we shall obtain the following expression

$$u_t = \frac{A_{10}}{X} \sqrt{\frac{1 - \mathfrak{K}^2 X^2}{\mathfrak{K}^2}} + \frac{2}{\pi} K_{10} \left(\arccos \sqrt{\frac{(1 + \mathfrak{K} L)(1 - \mathfrak{K} X)}{2 \mathfrak{K}(L - X)}} - \right. \quad (12)$$

$$\left. - \arccos \sqrt{\frac{(1 + \mathfrak{K} L)(1 + \mathfrak{K} X)}{2 \mathfrak{K}(L + \mathfrak{K})}} \right) + \frac{2}{\pi} \frac{q_t}{b} \int_0^{T_0} T \left(\operatorname{argch} \sqrt{\frac{(1 + \mathfrak{K} X)(1 - \mathfrak{K} T)}{2 \mathfrak{K}(X - T)}} + \right.$$

$$\left. - \operatorname{argch} \sqrt{\frac{(1 - \mathfrak{K} X)(1 + \mathfrak{K} T)}{2 \mathfrak{K}(X + T)}} \right) dT,$$

which, as a result of calculation, becomes:

$$u_t = \frac{A_{10}}{X} \sqrt{\frac{1 - \mathfrak{K}^2 X^2}{\mathfrak{K}^2}} + \frac{2}{\pi} K_{10} \arccos \mathfrak{K} L \sqrt{\frac{1 - \mathfrak{K}^2 X^2}{\mathfrak{K}^2(L^2 - X^2)}} +$$

$$+ \frac{q_t}{\pi b} \left[\frac{1}{2} (T_0^2 - X^2) \left(\operatorname{argch} \frac{1 - \mathfrak{K}^2 T_0 X}{\mathfrak{K}(X - T_0)} - \operatorname{argch} \frac{1 + \mathfrak{K}^2 T_0 X}{\mathfrak{K}(X + T_0)} \right) + \right.$$

$$\left. + \frac{X}{\mathfrak{K} b} \sqrt{1 - \mathfrak{K}^2 X^2} \arcsin \mathfrak{K} T_0 \right]. \quad (13)$$

Taking the equation (2) into consideration, we shall obtain the following expression of the axial perturbation velocity for the points on the wing ($x=y, s=z=0$).

$$u_{1a} = a_{10} \sqrt{\frac{1 - B^2 y^2}{\mathfrak{b}^2 + y^2}} + \frac{2}{\pi} K_{10} \arccos \sqrt{\frac{(l^2 + \mathfrak{b}^2)(1 - B^2 y^2)}{(1 + B^2 \mathfrak{b}^2)(l^2 - y^2)}} +$$

$$+ \frac{q_t}{\pi b} \left[\frac{1}{B} \sqrt{(1 - B^2 y^2)(\mathfrak{b}^2 + y^2)} \arccos \sqrt{\frac{1 - B^2 \tau_0^2}{1 + B^2 \mathfrak{b}^2}} - \right.$$

$$\left. \dots \right] \quad (14)$$

$$-(\tau_0^2 + y^2) \arg \operatorname{ch} \sqrt{\frac{(1+B^2\tau_0^2)(b^2+y^2)}{(1+B^2b^2)(\tau_0^2+y^2)}} \quad (14)$$

but on the surface of the sheet, where we observe that

$$\eta = y = 0, \quad x = i\zeta = \frac{iz}{\sqrt{1-B^2z^2}}, \quad (15)$$

we get

$$\begin{aligned} u_{lp} = a_{10} \sqrt{\frac{1-B^2h^2}{h^2-z^2}} + \frac{2}{\pi} K_{10} \arccos \sqrt{\frac{\lambda^2-h^2}{\lambda^2-z^2}} + \\ + \frac{q_t}{\pi h(1-B^2z^2)} \left(\frac{1}{B} \sqrt{\frac{h^2-z^2}{1-B^2h^2}} \arccos \sqrt{\frac{1-b^2\tau_0^2}{1-B^2\tau_0^2}} - \right. \\ \left. - \frac{\tau_0^2-z^2}{1-B^2\tau_0^2} \arg \operatorname{ch} \sqrt{\frac{h^2-z^2}{\tau_0^2-z^2}} \right), \quad \left(\lambda = \frac{t}{\sqrt{\sigma^2-1}} \right). \end{aligned} \quad (16)$$

2. Cruciform Wing of Symmetric Thickness with Slopes Equal to the Incidence of the First Component Wing

We shall introduce the double cruciform wing with a symmetrical thickness plate in order to remove the accentuated pressure apices on its intrados.

Proceeding in the same way as in the case of a thin wing, we shall obtain the following expression for the axial perturbation velocity in plane X:

$$\begin{aligned} u_t = \frac{2}{\pi} Q_{10} \arg \operatorname{ch} \frac{1}{\mathfrak{B}X} + \frac{2}{\pi} \frac{q_t}{b} \int_0^{\tau_0} T \left(\arg \operatorname{ch} \sqrt{\frac{(1+\mathfrak{B}X)(1-\mathfrak{B}I)}{2\mathfrak{B}(X-T)}} + \right. \\ \left. + \arg \operatorname{ch} \sqrt{\frac{(1+\mathfrak{B}X)(1-\mathfrak{B}I)}{2\mathfrak{B}(X+T)}} \right) dT, \end{aligned} \quad (17)$$

which becomes

$$\begin{aligned} u_t = \frac{2}{\pi} Q_{10} \arg \operatorname{ch} \frac{1}{\mathfrak{B}b} + \frac{q_t}{\pi b} \left[\frac{1}{2} (T_0^2 - X^2) \left(\arg \operatorname{ch} \frac{1-\mathfrak{B}T_0X}{\mathfrak{B}(X-T_0)} + \right. \right. \\ \left. \left. + \arg \operatorname{ch} \frac{1+\mathfrak{B}T_0X}{\mathfrak{B}(X+T_0)} \right) + X^2 \arg \operatorname{ch} \frac{1}{\mathfrak{B}X} + \frac{1}{\mathfrak{B}^2} \sqrt{1-\mathfrak{B}X^2} (1-\sqrt{1-\mathfrak{B}T_0^2}) \right]. \end{aligned} \quad (18)$$

The expressions of the axial perturbation velocities in the physical plane will be

$$\begin{aligned} u_{ta} = \frac{2}{\pi} Q_{10} \arg \operatorname{ch} \sqrt{\frac{1+B^2b^2}{B^2(b^2+y^2)}} + \frac{q_t}{\pi b} \left[(b^2+y^2) \arg \operatorname{ch} \sqrt{\frac{1-B^2b^2}{B^2(b^2+y^2)}} - \right. \\ \left. - (\tau_0^2+y^2) \arg \operatorname{ch} \sqrt{\frac{1+B^2\tau_0^2}{B^2(\tau_0^2+y^2)}} + \frac{1}{B^2} \sqrt{1-B^2y^2} (\sqrt{1-B^2b^2} - \sqrt{1+B^2\tau_0^2}) \right], \end{aligned} \quad (19)$$

on the wing surface, and

$$\begin{aligned} u_{tp} = \frac{2}{\pi} Q_{10} \arg \operatorname{ch} \sqrt{\frac{1-B^2z^2}{B^2(h^2-z^2)}} + \\ + \frac{q_t}{\pi h(1-B^2z^2)} \left[\frac{h^2-z^2}{1-B^2h^2} \arg \operatorname{ch} \sqrt{\frac{1-B^2z^2}{B^2(h^2-z^2)}} - \right. \\ \left. - \frac{\tau_0^2-z^2}{1-B^2\tau_0^2} \arg \operatorname{ch} \sqrt{\frac{1-B^2z^2}{B^2(\tau_0^2-z^2)}} + \right. \\ \left. + \frac{\sqrt{1-B^2z^2}}{B^2 \sqrt{(1-B^2h^2)(1-B^2\tau_0^2)}} (\sqrt{1-B^2\tau_0^2} - \sqrt{1-B^2h^2}) \right], \end{aligned} \quad (20)$$

on the plate surface.

3. Cruciform Wing of Symmetric Thickness Compensating for Slope

We shall compensate for the effect of the wing thickness resulting from the superposition of the first and second wings by introducing on the wing surface a new distribution of sources of a form which will return the wing to a mean zero thickness. Normal velocity variations on sheet v'' , created by a new distribution of origins, will correspond to a "wing compensating for a slope" of symmetric thickness.

Taking equation (11) into consideration, we shall choose for the points of this wing

$$q_i''(t) = k_i \frac{t}{h}, \quad (-h \leq t \leq h), \quad (21)$$

which in sheet X become

$$q_i''(T) = \frac{k_i}{\pi b} \sqrt{1 - \frac{b^2}{h^2}} \frac{d}{dT} \left(\frac{h^2 - T^2}{1 - \frac{b^2}{h^2} - T^2} \right), \quad (-b \leq T \leq b), \quad (22)$$

and we get the following expression of axial perturbation velocity:

$$\begin{aligned} u_c = & -\frac{2}{\pi} Q_{10} \arg \operatorname{ch} \frac{1}{\pi b} + \frac{2}{\pi} \frac{k_i}{\pi b} \sqrt{1 - \frac{b^2}{h^2}} \int_0^b \sqrt{\frac{h^2 - T^2}{1 - \frac{b^2}{h^2} - T^2}} \times \\ & \times \left(\arg \operatorname{ch} \sqrt{\frac{(1 + \frac{X}{b})(1 - \frac{T}{b})}{2 \frac{X}{b}(X - T)}} + \arg \operatorname{ch} \sqrt{\frac{(1 + \frac{X}{b})(1 + \frac{T}{b})}{2 \frac{X}{b}(X + T)}} \right) d \sqrt{\frac{h^2 - T^2}{1 - \frac{b^2}{h^2} - T^2}} = \\ & = -\frac{2}{\pi} Q_{10} \arg \operatorname{ch} \frac{1}{\pi X} - \frac{k_i}{\pi b} \sqrt{1 - \frac{b^2}{h^2}} \left[(1 - \frac{b^2}{h^2}) X^2 \arg \operatorname{ch} \frac{1}{\pi X} - \right. \\ & \quad \left. - (X^2 - b^2) \arg \operatorname{ch} \sqrt{\frac{1 - \frac{b^2}{h^2}}{B^2(X^2 - b^2)}} + \right. \\ & \quad \left. + \frac{1}{\pi^2} \sqrt{1 - \frac{b^2}{h^2}} (1 - \sqrt{1 - \frac{b^2}{h^2}}) \sqrt{1 - \frac{b^2}{h^2}} \right]. \end{aligned} \quad (23)$$

Calculating the axial perturbation velocities on wing and sheet we get:

$$\begin{aligned} u_{ca} = & -\frac{2}{\pi} Q_{10} \arg \operatorname{ch} \sqrt{\frac{1 + B^2 b^2}{B^2(h^2 + y^2)}} - \frac{k_i}{\pi b} \sqrt{1 + B^2 h^2} \frac{1}{1 - B^2 y^2} \times \\ & \times \left\{ (h^2 + y^2) \arg \operatorname{ch} \sqrt{\frac{1 + B^2 b^2}{B^2(h^2 + y^2)}} - (1 + B^2 h^2) y^2 \arg \operatorname{ch} \sqrt{\frac{1}{B^2 y^2}} + \right. \\ & \quad \left. + \frac{1}{B^2} \sqrt{1 + B^2 b^2} (\sqrt{1 + B^2 b^2} - 1) \sqrt{1 - B^2 y^2} \right\}, \end{aligned} \quad (24)$$

or

$$\begin{aligned} u_{cp} = & -\frac{2}{\pi} Q_{10} \arg \operatorname{ch} \sqrt{\frac{1 - B^2 z^2}{B^2(h^2 - y^2)}} - \\ & - \frac{k_i}{\pi h} \left[(h^2 - z^2) \arg \operatorname{ch} \sqrt{\frac{1 - B^2 z^2}{B^2(h^2 - z^2)}} + z^2 \arg \operatorname{ch} \sqrt{\frac{1 - B^2 z^2}{B^2 z^2}} + \right. \\ & \quad \left. + \frac{1}{B^2} (1 - \sqrt{1 - B^2 h^2}) \sqrt{1 - B^2 z^2} \right]. \end{aligned} \quad (25)$$

Observations

a) By superpositioning these three component wings we get a real wing for which the axial perturbation velocity is the expression:

$$u = u_1 + u_2 + u_3, \quad (26)$$

which will be antisymmetric to the axis of symmetry Ox_1 , continuous and different from zero at origin 0.

b) If we made $h \rightarrow 0$, ($h \rightarrow 0$), in the results obtained, we get the case of a delta wing with forced antisymmetry and with supersonic leading edges.

3. Determination of Constants

We calculate the constants Q_{10} , K_{10} , q_t , k_t in the same way as in a plane delta wing [3, 9].

Thus, by using some conditions limiting normal perturbation velocities, we find the constants a_{10} , K_{10} which appear in the expression of the axial perturbation velocity (13). These equations are found beginning with the compatibility equations

$$d u = -x d \varphi = \frac{i x}{\sqrt{1 - B^2 x^2}} d \varphi, \quad (27)$$

and considering the variations in velocity at one point on a wing or sheet, up to a point of zero velocity at the Mach cone. Likewise in the plane delta wing [3] we shall consider some concentrated sources at the point of their distribution with intensity Q_t and position $Y = T'_0$ given by the equations

$$Q_t = \frac{1}{2} \frac{q_t}{h} (h^2 - t_0^2), \quad (28a)$$

$$T'_0 = \frac{2}{3} T_0, \quad (28b)$$

which, written in the physical plane, become

$$Q_t = \frac{1}{2} \frac{q_t}{h} \frac{h^2 - t_0^2}{(1 - B^2 h^2)(1 - B^2 t_0^2)}, \quad (29a)$$

$$t_0 = \sqrt{\frac{9 h^2 (1 - B^2 t_0^2) - 4 (h^2 - t_0^2)}{4 (1 - B^2 h^2) + 5 (1 - B^2 t_0^2)}}, \quad (29b)$$

as a function of t_0 which limits the source distribution on the first thin sheet. We shall write the following equations:

$$Re B \int_{\text{strip 2}}^{\text{circular Mach 1}} \sqrt{\frac{1 - \mathfrak{B}^2 \lambda^2}{\mathfrak{B}^2 (h^2 - \lambda^2)}} d\mathfrak{U}'_t = -w, \quad (30a)$$

$$Re \int_{\text{plate 3}}^{\text{circular Mach 1}} \sqrt{\frac{1}{\lambda^2 - h^2}} d\mathfrak{U}'_t = v_1, \quad (30b)$$

(Key: 1- Mach Number; 2- Wing; 3- Sheet.)

where U'_t is the axial perturbation velocity of the first wing component in the case of sources concentrated in $z=t'$:

$$\begin{aligned} \mathfrak{U}'_t = & \pm \frac{A_{10}}{X} \sqrt{\frac{1 - \mathfrak{B}^2 X^2}{\mathfrak{B}^2}} + \frac{2}{\pi} K_{10} \left(\arccos \sqrt{\frac{(1 - \mathfrak{B} L)(1 - \mathfrak{B} X)}{2 \mathfrak{B} (L - X)}} - \right. \\ & - \arccos \sqrt{\frac{(1 + \mathfrak{B} L)(1 + \mathfrak{B} X)}{2 \mathfrak{B} (L + X)}} \Big) + \frac{2}{\pi} Q_t \left(\operatorname{argch} \sqrt{\frac{(1 + \mathfrak{B} X)(1 - \mathfrak{B} T_0)}{2 \mathfrak{B} (X - T_0)}} - \right. \\ & - \operatorname{argch} \sqrt{\frac{(1 + \mathfrak{B} X)(1 + \mathfrak{B} T_0)}{2 \mathfrak{B} (X + T_0)}} \Big) = \pm a_{10} \sqrt{\frac{1 - B^2 x^2}{b^2 + x^2}} \pm \\ & \pm \frac{2}{\pi} K_{10} \arccos \sqrt{\frac{(l^2 + h^2)(1 - B^2 x^2)}{(1 + B^2 h^2)(l^2 - x^2)}} \pm \frac{2}{\pi} Q_t \operatorname{argch} \sqrt{\frac{(b^2 + x^2)(1 + B^2 \tau_0^2)}{(\tau_0^2 + x^2)(1 + B^2 b^2)}}. \end{aligned} \quad (31)$$

Integrating (30a) on the real axis between the limits of $(0, \infty)$ and

(30b) on the imaginary axis between (h, ∞) , in the complex plane x , we obtain the constant

$$K_{10} = \frac{\alpha l U_\infty}{\sqrt{B^2 l^2 - 1}}, \quad (32)$$

and also the equation

$$\begin{aligned} \frac{2}{\pi} \frac{Q_t}{\tau_0^{12}} \sqrt{\frac{(b^2 - \tau_0^{12})(1 + B^2 \tau_0^{12})}{1 + B^2 b^2}} \left[K(k) - \frac{b^2}{b^2 - \tau_0^{12}} \Pi(q_2, k) \right] + \\ + \frac{2}{\pi} w \sqrt{\frac{l^2 + b^2}{1 + B^2 b^2}} \left[K(k) - \frac{b^2}{b^2 + l^2} \Pi(q_1, k) \right] = v_1. \end{aligned} \quad (33)$$

where the module k and the parameters ρ_1, ρ_2 of the complete elliptical integrals which appear are

$$k = \sqrt{1 - B^2 h^2}, \quad q_1 = -\frac{l^2}{l^2 + b^2}, \quad q_2 = \frac{\tau_0^{12}}{b^2 - \tau_0^{12}}. \quad (34)$$

From the condition of finite velocity at the subsonic edges of the sheets $(x = \pm i h)$ we reduce

$$a_{10} = A_{13} = 0. \quad (35)$$

The constant q_t is calculated by us by determining Q_t for the first time from equation (31). Starting with equation

$$q'_t(t) = \frac{t}{\sqrt{1 - B^2 t^2}} \frac{dv'}{dt}, \quad (36)$$

deduced from the theory of conical movements [1], and taking into consideration the fact that q'_t taken from (11) can also be written thus in plane x,

$$q'_t(t) = -\frac{q_t}{b} \frac{t}{(1 - B^2 t^2)^{3/2}}. \quad (37)$$

we will write the equations:

$$v_1 - v_0 = -\frac{q_t}{b} \int_{t_0}^b \frac{dt}{(1 - B^2 t^2)^{3/2}} \quad (38a)$$

$$v_0 t_0 + v' t \Big|_{t_0}^h - \int_{t_0}^h t dv' = v h. \quad (38b)$$

These equations were written by placing limiting conditions at points $t=t_0$ and $t=h$ for the lateral velocity v' , as well as the condition of real incidence in order to obtain the mean incidence.

As a result of calculations, we deduce from (38a) and (38b) the equations:

$$q_t (h \sqrt{1 - B^2 t_0^2} - t_0 \sqrt{1 - B^2 h^2}) = (v_0 - v_1) h \sqrt{1 - B^2 t_0^2}, \quad (39a)$$

$$q_t (\sqrt{1 - B^2 h^2} - \sqrt{1 - B^2 t_0^2}) = (v_1 - v_0) B^2 h^2 \sqrt{1 - B^2 t_0^2}. \quad (39b)$$

Next we introduce v_1 from (39b) and Q_t from (28a) and obtain the constant q_t in the following form:

$$\frac{q_t}{U_\infty} = - \frac{2 \sqrt{\frac{t^2 + b^2}{t^2 (1 + B^2 b^2)}} \left[K(k) - \frac{b^2}{t^2 + b^2} \Pi(\varrho_1, k) \right] \alpha - \beta}{\frac{9}{4} \left(\frac{b^2}{t_0^2} - 1 \right) \sqrt{\frac{(b^2 - t_0^2) (1 - B^2 t_0^2)}{b^2 (1 + B^2 b^2)}} \left[K(k) - \frac{b^2}{b^2 - t_0^2} \Pi(\varrho_2, k) \right] + \frac{\pi}{B^2 b^2} \left[\sqrt{1 + B^2 b^2} - \sqrt{1 + \frac{1}{4} B^2 (9 t_0^2 - 5 b^2)} \right]}. \quad (40)$$

Equation (39a) was used to determine the velocity v_0 on the sheet. The constant k_t found in expression (23) is determined by beginning with an equation similar to (38b) and writing the equation for measuring the normal velocities on the sheets of the three component wings, which will have a mean slope equal to $-v$:

$$-v h = \int_0^h v'' dt = v'' t \Big|_0^h - \int_0^h t dv''. \quad (41)$$

Equation (41), in consideration of (21), becomes

$$3 B^2 h^2 (v - v_1) = k_t [1 - (1 - B^2 h^2)^{3/2}], \quad (42)$$

which, united with (39a), determines the constant k_t :

$$k_t = \frac{3 q_t}{1 - (1 - B^2 h^2)^{3/2}} \left(1 - \sqrt{\frac{1 - B^2 h^2}{1 - B^2 t_0^2}} \right). \quad (43)$$

4. Distribution of Pressure and Aerodynamic Properties

Calculation of the coefficient of pressure on the wing and sheet is made by using the formula

$$C_p = -2 \frac{u}{U_\infty} = -\frac{2}{U_\infty} R_e u, \quad (44)$$

In which the expressions for axial perturbation velocities U_a or U_p , obtained from equation (26), are introduced in turn (figure 5).

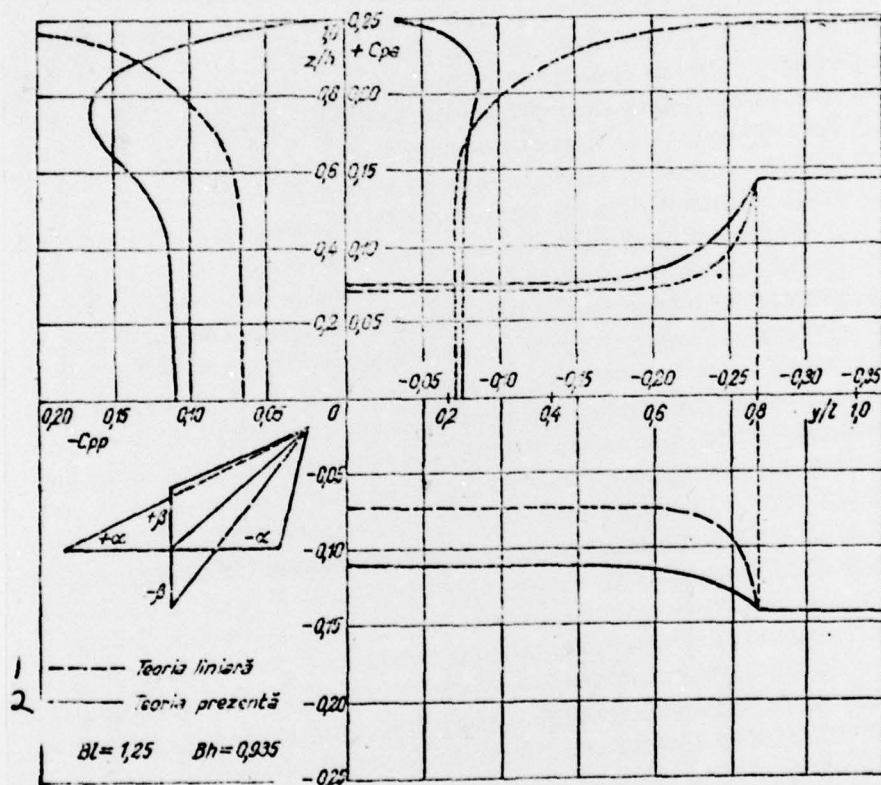


Figure 5. Key: 1-linear theory, 2-present theory

The coefficient of lift for both sheets or the wing is found by using formula [2]:

$$\frac{1}{2B} C_{zma} = \frac{2}{U_\infty} \int_0^{1.8} u_{1a} dy, \quad (45)$$

for the wing region comprised by the interior of the Mach cone,

$$\frac{1}{2} \left(l - \frac{1}{B} \right) C_{sca} = \frac{1}{2} \left(l - \frac{1}{B} \right) \frac{4\alpha}{B}, \quad (46)$$

for the outside region

$$C_{sp} = \frac{4}{h U_\infty} \int_0^h u_{lp} dz, \quad (47)$$

for the entire subsonic sheet and

$$\frac{1}{2} l C_{sa} = \frac{1}{2} \left(l - \frac{1}{B} \right) C_{sca} + \frac{1}{2B} C_{sma}, \quad (48)$$

for the entire horizontal wing.

The coefficient of moment of rolling is given by the following formulae:

$$H C_{ma} = \frac{8}{3 l U_\infty} \int_0^l u_{la} y dy, \quad (49)$$

where the horizontal plane and

$$H C_{mp} = \frac{8}{3 h U_\infty} \int_0^h u_{lp} z dz, \quad (50)$$

for the vertical plane, in which u_{la} and u_{lp} are given by (14) and (16).

In order to define the parameter $\frac{f_0}{h}$ we shall observe for the first time that the position of maximum pressure distribution coincides with that of the center of the vortex core, as is found by experimentation. On the other hand, by basing calculations on the distribution of selected sources we find that the apex of depression on the sheet extrados falls approximately in the center of gravity of the source intensity with position t' given by (29b).

To continue we shall use the formula

$$\frac{f_0}{h} = \frac{1}{1 + 1.7(\beta \pm \Delta\beta)^{1/2}}, \quad (51)$$

to define the position of the center of the vortex core in which β is the sheet incidence while $\Delta\beta$ is a supplementary incidence created by interference between the wing and the sheet, proportional to α . As a result, if $\alpha \rightarrow 0$, then $\Delta\beta \rightarrow 0$.

5. The Simplified Case of Concentrated sources

Assuming in a simplified way that the normal velocity at the surface of a sheet has a sudden jump into the center of a vortex core, equivalent to a sudden incident jump, we solve the problem from the hydrodynamic point of view by placing several concentrated sources at points t'_0 and $-t'_0$ of intensity Q_t and $-Q_t$.

The expressions of the axial perturbation velocities will be as follows: U

$$u'_t = \pm \frac{A_0}{X} \sqrt{1/B^2 - X^2} + \frac{2}{\pi} K_{10} \left(\operatorname{arccos} \sqrt{\frac{(1+B L)(1-B X)}{2 B(L-X)}} - \right. \\ \left. - \operatorname{arccos} \sqrt{\frac{(1+B L)(1-B X)}{2 B(L+X)}} \right) + \frac{2}{\pi} Q_t \left(\operatorname{argch} \sqrt{\frac{(1+B X)(1-B T_0)}{2 B(X-T_0)}} - \right. \\ \left. - \operatorname{argch} \sqrt{\frac{(1+B X)(1+B T_0)}{2 B(X+T_0)}} \right) = \pm a_{10} \sqrt{\frac{1-B^2 x^2}{b^2+x^2}} \pm \\ \pm \frac{2}{\pi} K_{10} \operatorname{arccos} \sqrt{\frac{(l^2+b^2)(1-\tau_0^2 x^2)}{(1+B^2 b^2)(l^2-x^2)}} \pm \frac{2}{\pi} Q_t \operatorname{argch} \sqrt{\frac{(b^2+x^2)(1+\tau_0^2 l^2)}{(\tau_0^2+x^2)(1+B^2 b^2)}}. \quad (52)$$

for a lifting cruciform wing,

$$u'_t = \frac{2}{\pi} Q_{10} \operatorname{argch} \frac{1}{B X} + \frac{2}{\pi} Q_t \left(\operatorname{argch} \sqrt{\frac{(1+B X)(1-B T_0)}{2 B(X-T_0)}} + \right. \\ \left. + \operatorname{argch} \sqrt{\frac{(1+B X)(1+B T_0)}{2 B(X+T_0)}} \right) = \pm Q_{10} \operatorname{argch} \sqrt{\frac{1+B^2 b^2}{b^2(b^2+x^2)}} \pm \\ \pm \frac{2}{\pi} Q_t \operatorname{argch} \sqrt{\frac{1+B^2 \tau_0^2}{B^2(\tau_0^2+x^2)}}. \quad (53)$$

for a wing of symmetric thickness, and

$$u'_c \equiv u_c, \quad (54)$$

for the third component wing.

The pressure distribution is found by substituting in (45) its expression U given by (26) in which U'_1 , U'_t , and U'_c come from (52), (53) and (54). The aerodynamic coefficients are found in the same way as in the case of distributed sources in which constants a_{10} , K_{10} , Q_t and k_t appear, deduced from equations

$$a_{10} = 0 \quad (55a)$$

$$v_0 l_0 + v_1(h - l_0) = v h, \quad (55b)$$

$$Q_t = \frac{(v_1 - v_0) l_0}{\sqrt{1-B^2 l_0^2}}, \quad (55c)$$

and from (33), and (42):

$$\frac{Q_t}{U_\infty} = - \frac{2 \sqrt{\frac{l^2+b^2}{l^2(1+B^2 b^2)}} \left[K(k) - \frac{b^2}{l^2+b^2} \Pi(q_1, k) \right] \alpha - \beta}{\frac{2}{\tau_0^2} \sqrt{\frac{(b^2-\tau_0^2)(1+B^2 \tau_0^2)}{1+B^2 b^2}} \left[K(k) - \frac{b^2}{l^2-\tau_0^2} \Pi(q_1, k) \right] -} \\ - \frac{1}{b} \sqrt{\frac{1+B^2 b^2}{1+B^2 \tau_0^2}}, \quad (56a)$$

$$\frac{k_t}{Q_t} = - \frac{3 F^2 h \sqrt{1 - B^2 t_0^{1/2}}}{1 - (1 - B^2 h^2)^{3/2}}, \quad (56b)$$

in which k , ρ_1 and ρ_2 are taken from (34). The constant K_{10} is the same as in (32).

Observations

a) The positions of the vortices are determined both from q and β , as is seen in (51).

b) If $\beta = 0$, $a_1 \neq 0$; the antisymmetric flow with vortices is again found.

c) Making $q_{10} = 0$ in the expression of axial velocity U in the linear theory, we get from the expression for a_{10} , calculated in [5], the condition as an antisymmetric cruciform wing in order to have finite velocities at the edge, avoiding the appearance of vortices:

$$\frac{\alpha}{\beta} = \frac{\pi l \sqrt{(l^2 + b^2)(1 + F^2 b^2)}}{2 [(l^2 + b^2) K(k) - b^2 \Pi(\varphi_1, k)]}. \quad (57)$$

From the same equation we deduce the supplementary incidence induced by the wing on the sheet when $\beta = 0$:

$$\Delta\beta = \frac{2 [(l^2 + b^2) K(k) - b^2 \Pi(\varphi_1, k)]}{\pi l \sqrt{(l^2 + b^2)(1 + B^2 b^2)}} \alpha, \quad (58)$$

introduced in (51).

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